MEMs Inertial Measurement Unit Calibration

1. Introduction

Inertial Measurement Units (IMUs) are everywhere these days; most particularly in smart phones and other mobile or handheld devices. These IMUs are typically constructed from MEMs[1] inertial sensors. Although the sensors are inexpensive they suffer from accuracy problems. This document describes a simple process for calibrating an IMU to improve its accuracy.

2. Errors in the sensors

An IMU consists of six individual sensors: three orthogonal angular rate gyros, and three orthogonal accelerometers[2].

2.1. Individual sensors

Each individual sensor is characterized with a linear equation:

1) $Output = (Measurement - Bias) \cdot Gain$

Where:

- *Output* is the data of interest in rad/s for the gyro, or m/s^2 for the accelerometer.
- *Measurement* is the raw reading from the sensor (in Volts or counts).
- *Bias* is the sensors offset in the same units as the measurement.
- *Gain* is the linear conversion from the raw reading to the output, with units (for example) of rad/s/Volt.

Each individual sensor has a sensitive axis, where load (acceleration or rate of rotation) on that axis produces the highest output, and load on the two orthogonal axes produce zero output.

2.2. Sensor triad cross coupling and alignment

Together the three gyros or three accelerometers are called a triad. Ideally the sensitive axes of the three sensors are perfectly orthogonal and aligned to the reference axes of the IMU. In reality manufacturing limitations prevent this ideal from being realized. Imagine an individual sensor (let's use the x-axis rate gyro as an example) whose physical alignment has the sensitive axis tilted towards the Y axis by an angle α and towards the Z axis by an angle β. This sensor would now give a measurement in response to rotation about Y and Z as well as X according to:

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[1] Micro Electromechanical Machines: small mechanical devices that are built using the same deposition and photolithographic techniques used to make integrated circuits.

[2] Despite their name accelerometers do not measure total acceleration. Like everything else an accelerometer cannot "feel" a body force such as gravity. Hence an accelerometer is actually a specific force sensor. One must add gravity to get the total acceleration.

2) $M_X - B_X = \frac{1}{G_X}(L_X \cos\theta_X + L_Y \sin\alpha_X + L_Z \sin\beta_X)$

Where:

- L_X, L_Y, L_Z are the true rotation rates about the X, Y, and Z reference axes (the load).
- M_X, B_X, G_X are the x-sensor measurement, bias, and gain respectively.
- α_X and β_X are the tilts of the sensitive axis towards the Y and Z reference axes respectively.
- Θ_X is the tilt of the x-sensor away from the X axis. $\cos\Theta_X$ is equal to $\sqrt{(1 - \sin^2\alpha_X - \sin^2\beta_X)}$.

If we divide the gain through to each term of the right hand side then equation 2) can be cast in matrix form and applied to all three axes:

3) $\overline{M} - \overline{B} = \boldsymbol{C}\overline{L}$

Where:

- $\overline{M}$ is the three sensor measurement vector
- $\overline{B}$ is the three sensor bias vector
- $\overline{L}$ is the X, Y, Z load vector (rotation or acceleration)
- $\boldsymbol{C}$ is the matrix $\begin{bmatrix} \frac{\cos\theta_X}{G_X} & \frac{\sin\alpha_X}{G_X} & \frac{\sin\beta_X}{G_X} \\ \frac{\sin\alpha_Y}{G_Y} & \frac{\cos\theta_Y}{G_Y} & \frac{\sin\beta_Y}{G_Y} \\ \frac{\sin\alpha_Z}{G_Z} & \frac{\sin\beta_Z}{G_Z} & \frac{\cos\theta_Z}{G_Z} \end{bmatrix}$

3. Applying the calibration

To convert a set of raw measurements from a triad to the final output the following matrix equation is applied:

4) $\overline{L} = \boldsymbol{A}(\overline{M} - \overline{B})$

Where:

- $\overline{L}$ is the best estimate of the true load vector (the true rates of rotation or specific forces) in the reference axes.
- ***A*** is a 3x3 matrix which is the inverse of ***C*** from equation 3).

The ***A*** matrix will be henceforth referred to as the *gain* matrix. This matrix contains the individual sensor gains, the cross coupling corrections, and the triad alignment rotation.

4. Determining the calibration

The problem then is to determine the bias vector $\overline{B}$ and the gain matrix ***A***.

4.1. Gyro calibration

The gyro calibration can be determined by rotating the IMU through a known set of rotations and recording the measurements. The choice of rotation order and magnitude is primarily driven by the mechanics of the device which does the rotation. In the simplest case a human may perform the rotation as long as the gyro triad is mounted in a square block, and a square corner is available to rotate against. In this instance the simplest set of rotations is 90^o in this order:

- π/2 radians[3] about body X (rotation 1)
- π/2 radians about body Y (rotation 2)
- π/2 radians about body Z (rotation 3)

Before the rotations the gyro triad is left at rest to determine the measurement with no load[4]. This gives the bias vector $\overline{B}$. During each rotation the measured rotation vector $\overline{R}$ is computed by time integrating the sensor measurements as:

5) $\overline{R} = \int (\overline{M} - \overline{B})\, dt$

Each rotation is used to fill out a column in the matrices of equation 6), which is simply an extended form of equation 4).

6) $\boldsymbol{L} = \boldsymbol{AR}$

Where:

- $\boldsymbol{L} = \begin{bmatrix} \frac{\pi}{2} & 0 & 0 \\ 0 & \frac{\pi}{2} & 0 \\ 0 & 0 & \frac{\pi}{2} \end{bmatrix} = \left| \overline{L}_1 \quad \overline{L}_2 \quad \overline{L}_3 \right|$
- $\overline{L}_1,\ \overline{L}_2,\ \overline{L}_3$ are the known load vectors of the rotations 1, 2, and 3.
- $\boldsymbol{R} = \begin{bmatrix} R_{X1} & R_{X2} & R_{X3} \\ R_{Y1} & R_{Y2} & R_{Y3} \\ R_{Z1} & R_{Z2} & R_{Z3} \end{bmatrix} = \left| \overline{R}_1 \quad \overline{R}_2 \quad \overline{R}_3 \right|$
- $\overline{R}_1,\ \overline{R}_2,\ \overline{R}_3$ are the integrated measurement vectors from the rotations 1, 2, and 3.
- ***A*** is the gain matrix we want to compute.

After all three rotations are performed every element of ***L*** and ***R*** is known and the ***A*** matrix is computed by solving equation 6) using matrix inverse:

7) $\boldsymbol{LR^{-1}} = \boldsymbol{A}$

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[3] Also known as 90^o.

[4] Strictly speaking the gyros are loaded according to the rotation of the Earth. However this rate (15 ^{o} per hour) is much smaller than typical MEMs gyros can detect; so we ignore it.

Note that the gain matrix ***A*** is the inverse of matrix ***C*** from equation 3). Hence taking the inverse of ***A*** allows you to determine the individual gain of each sensor by computing the inverse of the length of the x, y, or z row of ***C***. It also allows you to determine the exact angular misalignment of each sensor by inspection of the off diagonal terms of ***C***. However it is not necessary to actually compute ***C*** for purposes of calibration, unless you intend to allow the bias and gain to vary with temperature, as discussed in section 6.1.

4.2. Accelerometer calibration

The accelerometer calibration is done similarly to the gyro calibration, but there is an added difficulty. Determination of the bias of the gyros was easily done by observing the gyro measurements when the IMU was at rest. In the case of the accelerometers there is no unloaded state because the response to gravity[5] is always present[6].

To determine accelerometer bias when loaded by gravity the natural idea is to simply measure the output of the accelerometer and then subtract the effect due to gravity. However bias is in raw units (Volts or counts) and output is in acceleration (m/s^2). In order to convert between the two we need to know the gain, but we do not yet know that.

Therefore in order to compute the bias we need to make two measurements: in between the two measurements the accelerometer is inverted such that gravity affects the device in the opposite direction. The bias is then computed as:

8) $$B = (M_1 + M_2)/2$$

Where M_1 is the first measurement and M_2 is the second measurement performed after inverting[7] the accelerometer. In principle the starting orientation of the two measurements is irrelevant as long as the inversion requirement is met. However perfect inversion is hard to achieve, and the error due to this is a function of the starting orientation. The change in measurement due to a change in the tilt of the sensitive axis with respect to gravity is:

9) $$\frac{dM}{d\theta} = \frac{g}{G}\sin\theta$$

Where

- G is the gain of the sensor
- g is the acceleration due to gravity
- Θ is the angle between the sensitive axis and gravity

[5] This is often a confusing point. The accelerometers *do not* measure gravity. Since gravity is always acting on the accelerometer, some other *specific* force must be opposing gravity to keep the accelerometer from falling through the table. It is this specific force that the accelerometer measures. Hence the reading of an accelerometer at rest is the *opposite* of gravity.

[6] One could imagine measuring the accelerometer bias by going into orbit, but this seems cost prohibitive.

[7] We here define inversion as rotating half a circle about an axis which is orthogonal to gravity.

Equation 9) tells us that the effect of angular error during the inversion will be minimized when the sensitive axis is parallel to gravity (i.e. Θ is small). Therefore the orientation of the bias computation measurements should be done to maximize the loading due to gravity on the individual sensor. This implies that computing the bias for the entire triad will require six measurements in six orientations:

- A pair of measurements with the reference X axis pointing up and down (name these measurements X_{UP} and X_{DOWN}).
- A pair of measurements with the reference Y axis pointing up and down (name these measurements Y_{UP} and Y_{DOWN}).
- A pair of measurements with the reference Z axis pointing up and down (name these measurements Z_{UP} and Z_{DOWN}).

Although gravity is a complicating factor in bias determination it is very useful in determination of the gain matrix. Using our knowledge of the acceleration due to gravity we can use the UP and DOWN pairs of measurements to compute the gain, by taking the difference of the measurements. This is an approximation since the actual alignment of the sensitive axis with the load is not known. Nevertheless this approximation is quite good, within 1% for alignment errors as large as 8^{o}.

An even better method is available to us. If during the paired bias measurements we record the measurement for all three axes of the accelerometer triad we can fill out the elements of the ***L*** and ***R*** matrices of equation 6).

Where:

- $\boldsymbol{L} = \begin{bmatrix} 2g & 0 & 0 \\ 0 & 2g & 0 \\ 0 & 0 & 2g \end{bmatrix} = \left| \overline{L}_X \quad \overline{L}_Y \quad \overline{L}_Z \right|$
- $\overline{L}_X,\ \overline{L}_Y,\ \overline{L}_Z$ are the known load vectors that represent the difference in output between the UP and DOWN measurement pairs.
- $\boldsymbol{R} = \begin{bmatrix} M_{X,XUP} - M_{X,XDOWN} & M_{X,YUP} - M_{X,YDOWN} & M_{X,ZUP} - M_{X,ZDOWN} \\ M_{Y,XUP} - M_{Y,XDOWN} & M_{Y,YUP} - M_{Y,YDOWN} & M_{Y,ZUP} - M_{Y,ZDOWN} \\ M_{Y,XUP} - M_{Z,XDOWN} & M_{Z,YUP} - M_{Z,YDOWN} & M_{Z,ZUP} - M_{Z,ZDOWN} \end{bmatrix} = \left| \overline{R}_1 \quad \overline{R}_2 \quad \overline{R}_3 \right|$
- $\overline{R}_1,\ \overline{R}_2,\ \overline{R}_3$ are the differenced measurement vectors from the paired measurements X_{UP} and X_{DOWN}, Y_{UP} and Y_{DOWN}, Z_{UP} and Z_{DOWN}.
- ***A*** is the gain matrix we want to compute.

As with the gyro calibration the gain matrix ***A*** is computed according to equation 7).

5. Simultaneous calibration of gyros and accelerometers

Our goal is to compute the bias vector and gain matrix ***A*** for both gyros and accelerometers. For both gyros and accelerometers this involves moving the IMU through a series of rotations and separately recording the measurements for the periods at rest and while rotating. It is possible to combine these steps to simultaneously calibrate both triads.

There are many sequences of steps to accomplish this but one suggested sequence is this:

- Place the IMU in the Z_{UP} orientation, record the Z_{UP} accelerometer measurement and gyro bias vector.
- Rotate the IMU about the X axis 180 degrees to the Z_{DOWN} orientation. Record the integrated rotation 1 gyro measurements during the rotation. Record the Z_{DOWN} accelerometer measurement.
- Rotate the IMU about the Y axis 90 degrees to the X_{DOWN} orientation. Record the X_{DOWN} accelerometer measurement.
- Rotate the IMU about the Y axis 180 degrees to the X_{UP} orientation. Record the integrated rotation 2 gyro measurements during the rotation. Record the X_{UP} accelerometer measurement.
- Rotate the IMU about the Z axis 90 degrees to the Y_{DOWN} orientation. Record the Y_{DOWN} accelerometer measurement.
- Rotate the IMU about the Z axis 180 degrees to the Y_{UP} orientation. Record the integrated rotation 3 gyro measurements during the rotation. Record the Y_{UP} accelerometer measurement.

This sequence of rotations is different than the sequence that was outlined for the gyro calibration. However the different rotation sequence is easily accounted for by adjusting the values of the known load matrix ***L***.

6. Advanced topics

6.1. Value of gravity

The acceleration due to Earth's gravity is typically quoted as 9.806 m/s^2. This is an average number. The actual acceleration varies in a complex way according to the distribution of the Earth's mass. The gravity is primarily affected by the distance from the center of the Earth, which is greater at the equator than at the pole[8]. In addition the *apparent* gravity is even less at the equator because of the curving path of anything on the equator. The WGS84 Earth model gives a useful model for gravity as a function of latitude:

10) $$g(L) = 9.7803253359 \frac{1+0.001931853 sin^2 L}{\sqrt{1-e^2 sin^2 L}}$$

Where:

- *g(L)* is the gravity as a function of latitude in m/s^2.
- *L* is the latitude in radians.
- e^2 is 0.00669437999014, the square of the first eccentricity.

[8] Gravity at the equator is 9.78 m/s^2 and 9.83 m/s^2 at the poles. This variance of 0.05 m/s^2 is of similar order to the calibrated performance of a low cost accelerometer. Hence accounting for latitude during calibration is useful.

Gravity is also a function of height, and so for anyone calibrating an accelerometer on Mt. Everest you will want to scale your gravity calculation according to:

11) $g = g_{SL}\left(\frac{R_{EARTH}}{R_{EARTH}+h}\right)^2$

Where:

- g_{SL} is the gravity at sea level (as determined by equation 10).
- R_{EARTH} is the radius of the Earth. This is a function of latitude, but can be approximated as 6371000 meters.
- h is the altitude above sea level in meters.

6.2. Bias and gain variation with temperature

The bias and gain of most MEMs sensors are somewhat sensitive to temperature[9]. If a suitable robotic implementation of the rotation sequence is available then the calibration sequence can be performed repeatedly in a temperature chamber to determine the calibration as a function of temperature. Although bias and gain of the individual sensors are likely to vary over temperature the alignment and cross coupling should be much less sensitive. Therefore it may be useful to separate the gain from the ***A*** matrix. This reduces the amount of calibration information that must be stored and the amount of interpolation that is needed when applying the calibration to account for temperature.

6.3. Gyro bias variation with specific force

Up until now the bias of the gyro has been treated as a constant value that only needs to be observed once. However MEMs gyros may have some variation in bias due to specific force[10]. The calibration process can be modified to determine this bias variation and account for it. We model the gyro bias vector as the sum of two vectors.

12) $\overline{B} = \overline{B}_{ACCEL} + \overline{B}_{NOM}$

Where:

- $\overline{B}$ is the complete gyro bias vector
- $\overline{B}_{NOM}$ is the nominal gyro bias when no specific force is applied
- $\overline{B}_{ACCEL}$ is the additional bias due to specific force

[9] When the author first began working with low cost inertial sensors, in 1999, typical gyro bias variation over 100°C temperature change was 100s of deg/s. By 2003 the first generation of MEMs gyros had brought this down to 10s of deg/s. By 2014 the MEMs gyros in consumer devices are better than 5 deg/s. Accelerometers are simpler devices and do not suffer so much temperature sensitivity, but have also shown great improvement during this time period.

[10] The bias due to specific force can be as high as 0.2 deg/s per g. For high accelerations this can be significant. For most applications involving MEMs gyros this effect is small enough to be ignored.

When computing calibrated gyro measurements the process is to first compute the calibrated specific force measurement according to equation 4). The gyro bias due to specific force is then computed according to:

13) $\overline{B}_{ACCEL} = \boldsymbol{G}\overline{F}$

Where:

- $\overline{F}$ is the specific force vector after calibration has been applied to the specific force sensors.
- $\boldsymbol{G}$ is the gyro specific force sensitivity gain matrix

As before our calibration problem is to compute the matrix ***G***. During the static calibration measurements when the accelerometer measurements are recorded the gyro measurements are also recorded. This produces six sets of gyro bias measurements. The nominal gyro bias vector $\overline{B}_{NOM}$ is the average of these six measurements. We can use the paired differences of these measurements to fill out equation 14).

14) $\boldsymbol{R} = \boldsymbol{GL}$

Where:

- $\boldsymbol{L} = \begin{bmatrix} 2g & 0 & 0 \\ 0 & 2g & 0 \\ 0 & 0 & 2g \end{bmatrix} = \left| \overline{L}_X \quad \overline{L}_Y \quad \overline{L}_Z \right|$
- $\overline{L}_X$, $\overline{L}_Y$, $\overline{L}_Z$ are the known specific force load vectors that represent the difference in specific force between the UP and DOWN measurement pairs.
- $\boldsymbol{R} = \begin{bmatrix} M_{X,XUP} - M_{X,XDOWN} & M_{X,YUP} - M_{X,YDOWN} & M_{X,ZUP} - M_{X,ZDOWN} \\ M_{Y,XUP} - M_{Y,XDOWN} & M_{Y,YUP} - M_{Y,YDOWN} & M_{Y,ZUP} - M_{Y,ZDOWN} \\ M_{Y,XUP} - M_{Z,XDOWN} & M_{Z,YUP} - M_{Z,YDOWN} & M_{Z,ZUP} - M_{Z,ZDOWN} \end{bmatrix} = \left| \overline{R}_1 \quad \overline{R}_2 \quad \overline{R}_3 \right|$
- $\overline{R}_1$, $\overline{R}_2$, $\overline{R}_3$ are the differenced *gyro* measurement vectors from the paired measurements X_{UP} and X_{DOWN}, Y_{UP} and Y_{DOWN}, Z_{UP} and Z_{DOWN}.
- ***G*** is the matrix we want to compute.

The G matrix is then computed according to:

15) $\boldsymbol{RL^{-1}} = \boldsymbol{G}$

Once the ***G*** matrix is known each time a gyro vector measurement is taken an accelerometer vector measurement is also taken. The bias is then computed as:

16) $\overline{B} = \overline{B_{NOM}} + \boldsymbol{G}\overline{F}$

Where:

- $\overline{B}$ is the total gyro bias vector

- $\overline{B}_{NOM}$ is the nominal gyro bias vector computed as the average of the six gyro bias vectors that were taken in the three sets of paired UP/DOWN measurements.
- $\overline{F}$ is the specific force vector, which is computed from the raw accelerometer measurements and the accelerometer calibration according to equation 4) .

Now that the corrected gyro bias has been computed the rest of the gyro calibration routine can be performed. It is still possibly to simultaneously calibrate gyros and accelerometers, but this requires storing all the gyro measurements taken during rotation since the bias is not yet know at that time. It is simpler to just calibrate the accelerometers and gyro bias, and then do three more rotations to determine the gyro gain matrix.

7. About the author

Contact information:

Bill Vaglienti
Five by Five Development, LLC
Email: info@fivebyfivedevelopment.com
Website: http://www.fivebyfivedevelopment.com

Before founding Five by Five Development in 2013 Bill was co-founder of Cloud Cap Technology. While at Cloud Cap he (and partner Ross Hoag) developed the Piccolo flight management system, an extremely capable and popular proprietary FMS. Bill oversaw the software development of the Piccolo from its original concept into a market leading FMS. The Piccolo remains one of the most successful small UAV FMSs in use, with many thousands of units shipped and hundreds of thousands of hours of operation in military and civil applications.